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European Polymer Journal 42 (2006) 1836-1843



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# A solution for warpage in polymeric products by plug-assisted thermoforming

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Received 7 October 2005; received in revised form 13 March 2006; accepted 14 March 2006 Available online 22 May 2006

#### Abstract

Thermoforming process is one of the most popular techniques in polymer processing due to its interesting capabilities. The fundamental defect inherent to the thermoforming technology is warpage of the products during their application which becomes particularly apparent under high temperatures. The warpage is understood as the process of non-uniform (heterogeneous) change of the geometrical dimensions of products in time resulting in a change (distortion) of their original form. The results of this work specify the causes of the warpage and the conditions for its development. Thereby, it is possible to work out in providing valid recommendations for partial and, in some cases, complete elimination of the defect. © 2006 Elsevier Ltd. All rights reserved.

Keywords: Warpage; Plug-assist thermoforming; High temperature

## 1. Introduction

Polymer processing for production of all forms of polymeric articles has found a great place in chemical industries. Thermoforming process is one of the most popular techniques in this field. It applies to thermoplastic sheet as a film-forming technique for various packaging applications such as medical devices, food containers, and pharmaceuticals [1–4]. Wide applications of thermoforming are due to its high performance, simplicity, compactness and relatively low-cost equipment. These

issues make it possible to produce complex, largescale configurations and free form shapes of products. In thermoforming, a heated plastic sheet is stretched into a mold cavity by applying pressure and eventually direct mechanical loading are used [5,6]. Upon contact of a sheet with the cold surface of the mold, the sheet deformation is terminated. The forming sequence induces a thickness variation in the final product. Besides wall-thickness variation, other main problems facing the thermoforming industry are physical instabilities during inflation – rupture of sheet and shrinkage [7,8] exhibited in the final parts. There are many ways to stretch sheets: vacuum, air pressure and mechanical aids such as implementation of a plug. For increasing the quality of products such as narrow

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wall-thickness tolerance or elimination of frozen-in stresses, a combination of mechanical and vacuum or pressure forming methods can be implemented. The process initially involves the usage of mechanical pre-stretching with plug and then vacuum or pressure forming is applied. Fig. 1 illustrates this process.

Another fundamental and inherent defect to the thermoforming technology is warpage of the products during their application. This defect has particularly become apparent in high temperatures (Fig. 2). The warpage is understood as the process of non-uniform (heterogeneous) change of the geometrical dimensions in products within time domain. As a result it changes a product from its original form. Unfortunately, this problem is an overlooked area in thermoforming research.

The results of this work allow us to find out the causes of warpage and ascertain the conditions that rise to this defect. Thereby it is possible to provide valid recommendations for its partial or in some cases complete elimination.

## 2. Rheological model of plug-assist thermoforming

As mentioned above, shrinkage and warpage prediction is very important due to processing constraints. It may finally cause system failure. Proper description of the problem is directly dependent on the correct selection of an appropriate rheological model. Leonov developed a theoretical model in this area [9]:

$$\begin{split} & \bar{\sigma} + p\bar{\delta} = 2\bar{c}W_1 - 2\bar{c}^{-1}W_2 \\ & \bar{e}_{\mathrm{f}} = 1/\theta_0 G_0(T) \exp\{-\beta w^{\mathrm{s}}/G_0(T)\} \\ & \quad \cdot \left[ (\bar{c} - I_1\bar{\delta}/3)W_1^{\mathrm{s}} - (\bar{c}^{-1} - I_2\bar{\delta}/3)W_2^{\mathrm{s}} \right] \\ & \quad \frac{\mathrm{d}\bar{c}}{\mathrm{d}t} + \bar{\omega}\bar{c} - \bar{c}\bar{\omega} - \bar{c}(\bar{e} - \bar{e}_{\mathrm{f}}) - (\bar{e} - \bar{e}_{\mathrm{f}})\bar{c} = 0 \end{split} \tag{1}$$

where

 $\bar{\sigma}$  stress tensor

p Lagrange multiplier, determined by bound-

ary condition

 $\bar{\delta}$  identity tensor

 $\bar{c}$  Cauchy strain tensor

 $\bar{e}_{\rm f}$  flow strain rate tensor

 $\bar{\omega}$  vortex tensor

 $\bar{e}$  strain rate tensor

 $\theta_0(T)$  relaxation time

 $G_0(T)$  tensile modulus

W strain energy function  $W = W(I_1, I_2)$ 

 $\beta$  flexibility parameter of macromolecular

chains

 $I_1, I_2$  primary and secondary strain tensor invar-

iants

 $2W^{S} = W(I_1, I_2) + W(I_2, I_1)$  symmetric function of W

The last equation can be shown by:

$$W_1 = \frac{\partial W}{\partial I_1}, \quad W_2 = \frac{\partial W}{\partial I_2}$$
$$W_1^{S} = \frac{\partial W^{S}}{\partial I_1}, \quad W_2^{S} = \frac{\partial W^{S}}{\partial I_2}.$$

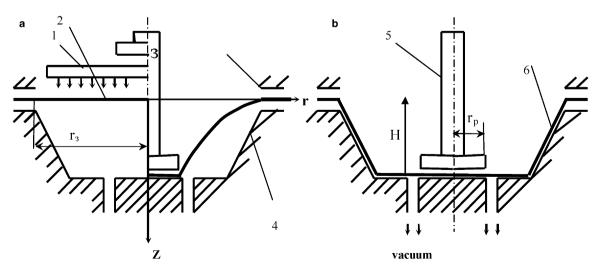


Fig. 1. Flow diagram of mechanical-vacuum thermoforming (a—heater and plug-assist; b—vacuum thermoforming): 1—heater, 2—sheet prior to forming, 3—clamp frame, 4—mold, 5—plug, 6—deformed sheet.



Fig. 2. The warpage of a polymeric product during its application.

In practice there is a problem for application of Eq. (1). This problem arises due to the choice of selecting strain energy function  $W = W(I_1, I_2)$ . Most researchers use Mooney–Rivlin potential, but there are inaccuracy in theoretical results for prediction of stress and strain and differ from the experimental ones. Results of recent research show that in various kinematical deformations, the following potential can be used [10]:

$$W = 0.25G_0(I_1 + I_2 - 6) (2)$$

By using the strain energy function presented in Eq. (2) in conjunction with the model in Eq. (1), a mathematical description for the deformation process occurring in mechanical pre-stretching mechanism of thermoforming, as shown in Fig. 1(a), can be developed.

## 3. Mathematical description

Analysis of the obtained results shows that the process of relaxation of residual (frozen-in) stress can cause warpage. This stress is accumulated during the thermoforming process. It can be proved by examination of an abstract task in description of the aftereffects with a deformation process on a viscoelastic polymer. It was preliminarily subjected to a plug with specified pure shear intensity. In this case, the kinetics of the development of general (viscoelastic) and elastic deformations resulting from application of a plug (mechanical pre-stretching) on the polymer. This issue is described in literature [1]. (Assuming that at a certain moment of time  $\tilde{t}_{\theta}$ , the mechanical pre-stretching was instantly ceased

by the removal of any external influence (plug) on polymer). Due to deformation, at this time, a certain level of general and elastic deformations is accumulated –  $\varepsilon^{\rm H}(\tilde{t}_{\phi}), \varepsilon^{\rm H}_{\rm e}(\tilde{t}_{\phi})$ . This condition corresponds to the "free" condition (not influencing from the outside) of the polymer in the molded product. Based on the rheological model (1), for a polymeric medium in a "free" condition, the relaxation process occurs at varying speeds as the level of accumulated elastic deformations reduces. This process is characterized by the following natural kinematic conditions:

$$\bar{e} = -\bar{e}_{\rm f} \tag{3}$$

The physical meaning of the last condition is that a polymer in absence of an external influence, the speed of deformation changes are determined only by the typical speed of the relaxation processes.

Using the condition (3) from the rheological model (1) for the examined and kinematically determined process of deformation aftereffects in the viscoelastic polymeric sheet, the following system of differential equations can be obtained that describes the kinetics of the process:

$$\begin{cases} \frac{\mathrm{d}c}{\mathrm{d}t} = -(c^2 - 1)\frac{1}{2\theta_0(T)} \exp[-\beta(c + c^{-1} - 2)] \\ \frac{\mathrm{d}\varepsilon^{\mathrm{H}}}{\mathrm{d}t} = -(c - c^{-1})\frac{1}{4\theta_0(T)} \exp[-\beta(c + c^{-1} - 2)] \end{cases}$$
(4)

where

$$\begin{split} c &\equiv \lambda_{\rm e}^2 = \exp(2\varepsilon_{\rm e}^{\rm H}), \\ \varepsilon^{\rm H} &\quad \text{Hencky strain} \end{split}$$

 $\varepsilon_{\rm e}^{\rm H}$  elastic Hencky strain  $\lambda_{\rm e}$  elastic strain ratio

The initial conditions for solving the system of ordinary differential equation (4),  $\varepsilon^{\rm H}(\tilde{t}_{\phi})$  and  $\varepsilon^{\rm H}_{\rm e}(\tilde{t}_{\phi})$ , are determined by deformation influences on the polymer. From the analysis of Eq. (4) it can be found that during deformation, the accumulated elastic deformations ( $\varepsilon^{\rm H}_{\rm e}$ ) influence on the polymer relax (the first equation) with varying speed until the removal of the applied mechanical field (plug) on the polymer. This leads to the process of a reduction in accumulated general deformations (second equation) which is going to be proved.

For rigid-chain polymers  $(\beta \to 0)$  or for a small level of accumulated elastic deformations  $(\exp(\epsilon_e^H(\tilde{t}_{\Phi})) \to 1)$ , the solution of ordinary differential equations systems (4) is shown as follows:

$$\begin{cases} \varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t}) \approx \frac{1}{2} \ln \frac{\frac{1 + \exp(2\varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t}_{\phi}))}{1 - \exp(2\varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t}_{\phi}))} \exp[(\tilde{t} - \tilde{t}_{\phi})] - 1}{\frac{1 + \exp(2\varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t}_{\phi}))}{1 - \exp(2\varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t}_{\phi}))} \exp[(\tilde{t} - \tilde{t}_{\phi})] + 1} \\ \varepsilon^{\mathrm{H}}(\tilde{t}) \approx \varepsilon^{\mathrm{H}}(\tilde{t}_{\phi}) - \frac{1}{4} \int_{\tilde{t}_{\phi}}^{\tilde{t}} [\exp(2\varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t})) - \exp(-2\varepsilon_{\mathrm{e}}^{\mathrm{H}}(\tilde{t}))] d\tilde{t} \end{cases}$$

$$(5)$$

where  $\tilde{t} = \frac{t}{\theta_0(T)}$  is dimensionless time,  $\tilde{t}_{\Phi}$  is dimensionless time of plug-assist forming,

Fig. 3 represents the kinetics of the free relaxation process of elastic deformations within viscoelastic polymers resulting in the parallel process of changing original dimensions.

#### 4. Results and discussion

The above statement clearly proves that the cause of molded products warpage during their application is accumulation of elastic deformations within the polymer. Since the accumulated elastic deformations in polymer can be distributed non-uniformly, this results in different speeds for the relaxation process. This results in noticeably different speeds of dimensional change in various parts of the product, i.e. in the warpage which is evidenced by Eq. (5). Thus, on the basis of this analysis one may state that the warpage originates under the condition of accumulation of heterogeneous elastic deformations in the polymer during the process of thermoforming. This issue manifests itself through the relaxation process of these deformations.

Gained equations of (5) establish this subject that the process of warpage is essentially affected by one of the polymers' fundamental rheological characteristics, namely typical relaxation time,  $(\theta_0(T))$ , that is temperature-dependent. By reducing the required time for the relaxation at the expense of increasing in the polymer's temperature, the relaxation processes in polymers run faster. Thus the speed of warpage consequently increases in such conditions.

Warpage of an item can be characterized quantitatively. For this subject, a coefficient for dimensional change is specified. It integrally characterizes the relative change in the original dimension of a product which can be expressed in following form:

$$k_{u\Phi}(\tilde{t}) = 1 - \frac{S(\tilde{t})}{S(\tilde{t} = \tilde{t}_{\Phi})} \tag{6}$$

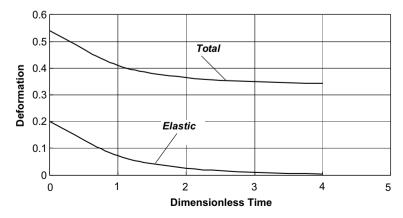


Fig. 3. The kinetics of the free relaxation process of elastic deformations in viscoelastic polymers resulting in the process of changing its original dimensions.

where  $S(\tilde{t}), S(\tilde{t} = \tilde{t}_{\Phi})$  are respectively the current area of the lateral surface of the distorted article and original area of the lateral surface of the product.

By using well-known equations and the mathematical results in this work  $(S(t) = 2\pi \int_0^H r(z) \sqrt{1 + (\frac{dr}{dz})^2} dz)$ , the following determining expression is obtained:

$$S(\tilde{t} = \tilde{t}_{\Phi}) = 2\pi r_3 \sqrt{b} \int_0^{\tilde{H}} \frac{\sqrt{(\sqrt{b} + \tilde{z})^4 + b}}{(\sqrt{b} + \tilde{z})^3} d\tilde{z}$$
 (7)

$$S(\tilde{t}) = 2\pi r_3 \sqrt{b(\tilde{r}_0 = 1, \tilde{t})} \int_{\tilde{r}_0 = \tilde{r}_p}^{\tilde{r}_0 = 1} \frac{\exp(\varepsilon^{H}(\tilde{r}_0, \tilde{t}))}{\tilde{r}_0} d\tilde{r}_0$$
 (8)

where

$$\begin{split} b(\tilde{t} = \tilde{t}_{\Phi}) &\equiv \lambda^2(\tilde{z} = 0) - 1 = \left(\tilde{H} \cdot \frac{\tilde{r}_{\mathrm{p}}}{1 - \tilde{r}_{\mathrm{p}}}\right)^2; \\ \tilde{r}_{\mathrm{p}} &\equiv \frac{r_{\mathrm{p}}}{r_{3}}; \quad \tilde{H} \equiv \frac{H}{r_{3}}; \quad \tilde{z} \equiv \frac{z}{r_{3}}; \quad \tilde{r}_{0} = r_{0}/r_{3} \\ b(\tilde{r}_{0} = 1, \tilde{t}) &\equiv \lambda^2(\tilde{r}_{0} = 1, \tilde{t}) - 1 = \exp(2\varepsilon^{\mathrm{H}}(\tilde{r}_{0} = 1, \tilde{t})) - 1. \end{split}$$

 $r_{\rm p}$  is plug radius;  $r_{\rm 3}$  is initial radius of sheet;  $r_{\rm 0}$  is current radius of sheet during plug-assist; r,z are the horizontal and vertical axes, respectively; H is the depth of deformed sheet.

The function  $\varepsilon^{H}(\tilde{r}_{0}, \tilde{t})$  is specified by the solution of the differential equations system (4).

By using the expressions (7) and (8), for Eq. (6), the required equation for describing of the kinetics of the warpage in the product is obtained. The data obtained from the developed expression for coefficient of dimensional change in conjunction with experienced data is presented in Fig. 4.

By assigning an allowable tolerance value for dimensional change coefficient of a product ( $[k_{\mu\phi}]$ ),

in a specific application, the maximum permissible time can be determined based on the following condition:

$$[k_{u\Phi}] = k_{u\Phi}(\tilde{t}) \tag{9}$$

where the function  $k_{u\Phi}(\tilde{t})$  is determined in Eq. (6).

Based on the analysis of specified condition in Eq. (9), it becomes clear that the application period of a polymeric product under given temperature conditions with the allowable coefficient of dimensional change is essentially determined by the accumulated elastic deformations  $(c(r_0, \tilde{t}_{\Phi}))$ , in the product during its formation process. The smaller elastic deformations accumulated mean longer operational period of the product. Based on this concern, a completely practical task for organizing the plug-assist vacuum-forming process arises, so that the total level of elastic deformations (i.e., the accumulated deformations during stages of the article's molding) in polymeric materials must be minimal. As the molding technology is consisted of two stages, it is necessary to analyze each of the

It is well known that the stage of vacuum-forming with a pre-stretched sheet occurs too quickly for experiencing of relaxation processes in the polymeric sheet. This will result in this fact that almost all accumulated deformations of polymeric materials in this stage is elastic. Consequently, the minimization of the accumulation may be realized only by minimizing of the general deformations accumulated in polymeric sheet during this stage. For practical purposes, this means that the profile of the prestretched sheet should be maximally approximated by the profile of the final product. This could be based on the application of a plug with a respective radius.

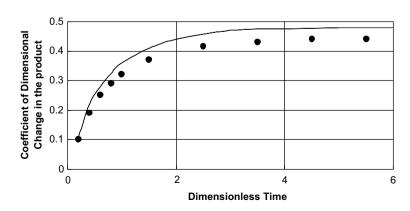


Fig. 4. The kinetics of the warpage of HIPS at T = 353 K,  $\theta_0(T) = 3$  s, (curve-Eq. (6) and points-experimental data).

In contrast to the second stage, the stage of mechanical pre-stretching by a plug can be regulated in the sense that it is technically possible to control the motion of the plug. This creates a practical opportunity that at this stage to organize the relaxation process of elastic deformations accumulated in a polymeric sheet during the process of mechanical pre-stretching. The essence of the simplest of many variants used in the realization of this process lies in the creation of a relaxation pause period between the first and the second stages. During this period the accumulated elastic deformations are partially or completely relaxed. The only problem may arise here is that during this pause period the pre-stretched sheet would not have time to cool to a certain temperature. This makes an invalid realization for the second technological stage of the thermoforming process. Evaluation of the necessary duration for this pause, ensure the realization of the relaxation processes.

During the relaxation pause, a plug holds a stretched sheet, one of the kinematical conditions of this process corresponds to the absence of external deforming influence is in the following form:

$$\bar{e} = 0 \tag{10}$$

By using the condition (10) in rheological model (1) for the kinematically determined process of the relaxation in a viscoelastic polymeric medium, the following equations can be obtained. This describes its kinetics:

$$\begin{cases} \frac{\mathrm{d}c}{\mathrm{d}t} = -\frac{1}{2}(c^2 - 1)\frac{1}{\theta_0(T)} \exp[-\beta(c + c^{-1} - 2)] \\ \frac{\mathrm{d}\varepsilon^{\mathrm{H}}}{\mathrm{d}t} = 0 \end{cases}$$
(11)

Fig. 5 presents the kinetics of change for elastic deformations in a sheet stretched by a plug during the relaxation pause.

By comparison of the differential equation systems (11) and (4), it is not difficult to conclude that the relaxation processes in polymers with limited conditions and free aftereffects are distinguished by speed.

This is quite logical to consider that the limited conditions of the relaxation aftereffect do not completely correspond to the removed mechanical field that superimposed on a polymeric media which also slows down the development of the relaxation processes. For rigid-chain polymers,  $(\beta \to 0)$ , or at a moderate level of the accumulated elastic deformations,  $(\exp(\varepsilon_{\rm e}^{\rm H}(r_0,t_{\Phi}))\to 1)$ , the analytical solution of equations system (11) is in following form:

$$\begin{cases} \varepsilon_{\mathbf{e}}^{\mathbf{H}}(r_{0},\tilde{t}) \approx \frac{1}{2} \ln \frac{1 + \exp\left(2\varepsilon_{\mathbf{e}}^{\mathbf{H}}(r_{0},\tilde{t}_{\Phi})\right)}{1 - \exp\left(2\varepsilon_{\mathbf{e}}^{\mathbf{H}}(r_{0},\tilde{t}_{\Phi})\right)} \exp[(\tilde{t} - \tilde{t}_{\Phi})] - 1}{\frac{1 + \exp\left(2\varepsilon_{\mathbf{e}}^{\mathbf{H}}(r_{0},\tilde{t}_{\Phi})\right)}{1 - \exp\left(2\varepsilon_{\mathbf{e}}^{\mathbf{H}}(r_{0},\tilde{t}_{\Phi})\right)} \exp[(\tilde{t} - \tilde{t}_{\Phi})] + 1} & (\tilde{t} \geqslant \tilde{t}_{\Phi}) \\ \varepsilon^{\mathbf{H}}(r_{0},\tilde{t}) = \varepsilon^{\mathbf{H}}(r_{0}) = \text{const} \end{cases}$$

$$(12)$$

The analysis of numerical solution of differential equation (11) for the most common cases shows that even in the most unfavorable conditions,  $(\beta > 1)$  and  $c(t_{\Phi} = 0) \approx 3$ , the accumulated elastic deformations will relax completely within a period of time that does not exceed from  $9\theta_0(T)$ , approximately. By considering this fact that for most thermoplastic materials at range of average recommended processing temperature, the typical relaxation time will be about in range of 0.2–0.5 s, this could be concluded that even in the most

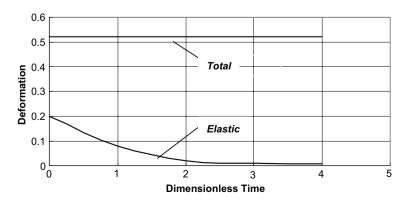


Fig. 5. The kinetics of the relaxation process of elastic deformations in viscoelastic polymer during the relaxation pause.

unfavorable situations, the time necessary for the relaxation processes in polymers would not exceed 4.5 s.

From [11] it is not difficult to determine that, during this time period, a polymeric sheet with a certain thickness, e.g. 1.0 mm, at a temperature of T=140 °C is cooled down only about 10 °C (thermodynamic figures of similarity of the examined conditions of cooling:  $Bi \approx 0.04$  and  $Fo \approx 1.75$ ). This reduction in temperature is not critical for the second stage of product formation even considering the time spent on the stage of mechanical pre-stretching.

For thinner sheets, obviously the maximum relaxation pause period becomes critical. But this criticality issue arise only under very strict requirements concerning the quality of the final products according to the admissible criterion of their distortion,  $[k_{u\Phi}] \rightarrow 0$ . In case of less strict conditions for permitting some level of distortion which its value can be determined by the value of the allowable coefficient ( $[k_{u\Phi}]$ ), the required technological time for the relaxation pause can be determined.

In this case, the minimum duration of the relaxation pause period should be a value that guarantees an infinitely large application time of a product with the allowable coefficient  $[k_{u\phi}]$ . Nominally, by considering Eq. (6), this may be expressed as follows:

$$S(\tilde{t} \to \infty) \geqslant S(\tilde{t}_{\Phi}) \cdot (1 - [k_{u\Phi}])$$
 (13)

By analysis of Eq. (8) it is clear that the function  $S(\tilde{t})$  is determined by solving of differential equations system (4). Consequently, solving the equations under different initial conditions of  $\varepsilon^{H}(r_0)$  and  $\varepsilon^{H}_{e}(r_0, \tilde{t}_{\Phi})$ , it is always possible to find such a solution

would critically satisfy the condition (13). Based on the results obtained in this work, the following two equations determine the initial conditions.

$$\varepsilon_{e}^{H}(\tilde{r}_{0}, \tilde{t}_{\phi}) = \frac{1}{2} \ln \left\{ \frac{1}{2} \left[ \frac{c_{0}^{2}(\tilde{t}_{\phi}) - 1}{c_{0}(\tilde{t}_{\phi})} \cdot \frac{1}{1 + b(\tilde{t}_{\phi})} \cdot \frac{\tilde{r}_{0}^{4} + b(\tilde{t}_{\phi})}{\tilde{r}_{0}^{3}} + \sqrt{\left(\frac{c_{0}^{2}(\tilde{t}_{\phi}) - 1}{c_{0}(\tilde{t}_{\phi})} \cdot \frac{1}{1 + b(\tilde{t}_{\phi})} \cdot \frac{\tilde{r}_{0}^{4} + b(\tilde{t}_{\phi})}{\tilde{r}_{0}^{3}} \right)^{2} + 4} \right] \right\} \\
\varepsilon^{H}(r_{0}) = \frac{1}{2} \ln \left[ \left(\frac{\tilde{H}\tilde{r}_{p}}{(1 - \tilde{r}_{p})\tilde{r}_{0}^{2}}\right)^{2} + 1 \right] \tag{14}$$

Based on Eq. (14), the procedure for solution of equations system (4) which critically satisfy the condition in Eq. (13) is actually reduced to a search for finding the critical (maximum) value of the elastic deformation,  $(c_{*0}(\tilde{t}_{\Phi}))$ . In the end of the relaxation pause period, the value of elastic deformation should not be exceeded in a certain section of a stretched sheet. This is determined to fulfill the presented condition in expression (13). As soon as the value  $c_{*0}(\tilde{t}_{\Phi})$  is determined, the duration of the necessary relaxation pause can be easily determined by solving the first differential equation (11). For the special case that is specified in above, substituting the obtained value as current and corresponding to the end of the relaxation pause period in the first equation of the equations system (12), an expression for determining the operational factors of the relaxation pause period is obtained. This ensures the realization of the relaxation process of elastic deformations to the required level:

$$t_{\rm pn} \approx \theta_0(T) \ln \left\{ \frac{c_0(\tilde{t}_{\Phi}) - 1}{1 + c_0(\tilde{t}_{\Phi})} \cdot \frac{1 + c_{*0}(\tilde{t}_{\Phi})}{c_{*0}(\tilde{t}_{\Phi}) - 1} \right\}$$
(15)

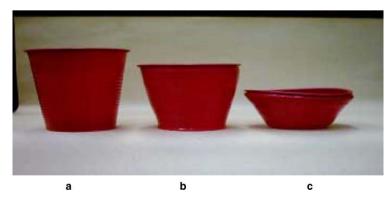


Fig. 6. Comparison of warpage in PS products (a—initial product at T = 298 K without warpage, b—at T = 353 K with  $t_{pn} = 2$  s during production process, c—at T = 353 K without relaxation pause during production process.

Confirmation of results of this model is exhibited in Fig. 6.

As expected, under the same conditions, the warpage of a product without a relaxation pause is much greater than the warpage of a product with a relaxation pause. This issue shows the importance of a relaxation pause in reducing warpage of products during their application, especially under high temperature conditions.

Based on the results obtained in this work, another method for reducing the level of elastic deformations of sheet stretching is developed. It calculates the lower speed level for plug movement. However, in this case the time of sheet stretching exceeds the total duration time on its stretching due to plug movement at the normal speed and the involved relaxation pause.

This is quite evident that in the first stage a more active mechanical force is imposed to the polymeric materials than the relaxation pause period. Consequently, the speed of the relaxation processes in the first stage is less than in the second stage. This should also be mentioned that the time necessary for the relaxation processes in the relaxation pause period can be reduced by increasing the sheet temperature. However, it must be noted that for removing of the defect, the temperature range of  $(T_{\rm g} < T < T_{\rm m})$  specifies a practical limit on operating condition for the examined technology.

#### 5. Conclusion

Analysis of the causes and conditions for occurrence of the products warpage conducted to develop a method for calculation of operational conditions of molding process. The operational conditions guarantee production of final products within acceptable warpage values. Furthermore, a method for calculating of the maximum production period of polymeric products with permitted warpage values is developed.

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